

Workshop on Harmonic Analysis and its Applications

on the occasion of Filippo De Mari's 65th birthday

Department of Mathematics, University of Genoa
26/27 September 2024

SCHEDULE

26 September (room 509)

- 14:15 Opening
- 14:30 Anita Tabacco – *Filippo: A Short Journey Through Mathematics and Academia*
- 15:05 Stephan Dahlke – *My Friendship with Filippo: History and Scientific Output*
- 15:40 Gabriele Steidl – *Slicing versus Fourier Random Features*
- 16:15 Coffee break
- 16:45 Marco Maria Peloso – *Explicit fundamental solutions of Kohn Laplacians, sub-Laplacians and higher order invariant differential operators*
- 17:20 Demetrio Labate – *Efficient clustering on Riemannian manifolds by Fréchet mappings*

27 September (room 508)

- 09:00 Michael Cowling – *Quasiconformal mappings on nilpotent Lie groups*
- 09:35 Alessandro Ottazzi – *CR embeddings and applications*
- 10:10 Maria Vallarino – *Dyadic sets in spaces of exponential growth*
- 10:45 Coffee break
- 11:15 Emilio Musso – *Integrable flows on Legendrian Curves in the 3-dimensional anti-de-Sitter space*
- 11:50 Claudio Bartocci – *Moduli spaces of quiver representations*
- 12:25 Marco Pedroni – *Poisson quasi-Nijenhuis manifolds and Toda lattices*

TITLES AND ABSTRACTS

Anita Tabacco (Politecnico di Torino)

Filippo: A Short Journey Through Mathematics and Academia

This talk explores the life and career of Filippo and highlights his significant contributions to the field of mathematics, his leadership roles in shaping academic programs, and his enduring impact as an inspiring educator and mentor.

Stephan Dahlke (Philipps-Universität Marburg)

My Friendship with Filippo: History and Scientific Output

In this talk, we will be concerned with one joint scientific project with Filippo, that is, with coorbit space theory. We start with a brief introduction of the basic concepts of coorbit theory. Then, we will discuss one of our contributions, that is, the generalization of the theory to the case on non-integrable kernels. Some examples will also be presented.

Gabriele Steidl (Technische Universität Berlin)

Slicing versus Fourier Random Features

Computations in high-dimensional spaces can often be realized only approximately, using a certain number of projections onto lower dimensional subspaces or sampling from distributions. In this paper, we are interested in pairs of real-valued functions (F, f) on $[0, \infty)$ that are related by the projection/slicing formula $F(\|x\|) = E_{\xi} [f(\|\langle x, \xi \rangle\|)]$ for $x \in \mathbb{R}^d$, where the expectation value is taken over uniformly distributed direction in S^{d-1} . While it is known that F can be obtained from f by an Abel-like integral formula, we construct conversely f from given F using their Fourier transforms. First, we consider the relation between F and f for radial functions $F(\|\cdot\|)$ that are Fourier transforms of L^1 functions. Besides d - and one-dimensional Fourier transforms, it relies on a rotation operator, an averaging operator and a multiplication operator to manage the walk from d to one dimension in the Fourier space. Then, we generalize the results to tempered distributions, where we are mainly interested in radial regular tempered distributions. Based on Bochner's theorem, this includes positive definite functions $F(\|\cdot\|)$.

Further, we are interested in choosing appropriate direction in S^{d-1} and deal with different QMC sequences and kernels.

This is joint work with Johannes Hertrich (UCL), Tim Jahn, Nikolaj Rux and Michael Quellmalz (TU Berlin).

References:

J. Hertrich, Fast kernel summation in high dimensions via slicing and Fourier transforms, SIAM J. Data Science, to appear

N. Rux, M. Quellmalz, G. Steidl, Slicing of radial functions: a dimension walk in the Fourier space, ArXiv Preprint 2024

J. Hertrich, T. Jahn, M. Quellmalz, Fast kernel summation via QMC slicing, ArXiv Preprint comes September 2024

Marco Maria Peloso (Università di Milano)

Explicit fundamental solutions of Kohn Laplacians, sub-Laplacians and higher order invariant differential operators

In the D.P.R. of 1995 an explicit and powerful formula was proved. In this talk, I want to draw the attention to such formula, to show its significance, indicate applications, possible generalizations, and ultimately celebrate Filippo's quest for beauty.

Demetrio Labate (University of Houston)

Efficient clustering on Riemannian manifolds by Fréchet mappings

Symmetric Positive Definite (SPD) matrices, in particular correlation matrices, appear in various applications from applied mathematics and engineering, most notably in neuroimaging applications such as Diffusion Tensor Imaging and Functional Magnetic Resonance Imaging, where they are employed to model the strength of neural connections between different brain sites. Mathematically, the set of SPD matrices is not a vector subspace of the Euclidean space under standard matrix addition and scalar multiplication, however it possesses a smooth manifold structure that can be endowed with a Riemannian metric. Hence the proper measure of similarity between SPD matrices is not a Euclidean distance but a Riemannian distance that can capture the intrinsic geometrical structure of the underlying space. Unfortunately, computation of distances in the Riemannian manifold of SPD matrices becomes very expensive as the matrix size increases so that computing k-means of large SPD matrices directly in the Riemannian setting can be prohibitively expensive. Here we present and demonstrate a novel approach to efficiently cluster data on the space of SPD matrices taking advantage of a specially designed Fréchet mapping. Joint work with R. Azencott, N. Charon, A. Mang and J. Shi.

Michael Cowling (University of New South Wales)

Quasiconformal mappings on nilpotent Lie groups

In his wayward youth, Filippo de Mari considered conformal mappings of Iwasawa N groups and more generally nilpotent Lie groups, with a view to classifying them. This talk aims to survey some of the major developments in this field in the last few decades.

Alessandro Ottazzi (University of New South Wales)

CR embeddings and applications

In this seminar, I summarise some recent results obtained with different collaborators on CR embeddings of nilpotent Lie groups and some quotients. As an application, a family of hypersurfaces in C^2 with no

homogenous structure is lifted to stratified Lie groups, optimising (at least in this case) a well-known procedure for bracket generating families of vector fields introduced by Rothschild and Stein in 1976.

Maria Vallarino (Politecnico di Torino)

Dyadic sets in spaces of exponential growth

In this talk we will start recalling the definition and the main properties of the family of classical dyadic cubes in the Euclidean space R^d , considering the dyadic maximal Hardy–Littlewood operator and the dyadic Calderón–Zygmund theory in this setting (see [3]). Thanks to a seminal paper by Christ [1], it is known that a family of dyadic sets can be constructed in doubling measure metric spaces, and a corresponding dyadic Calderón–Zygmund theory can be considered.

On nonhomogenous trees equipped with locally doubling flow measures, which are a class of graphs with exponential growth, a family of dyadic sets can be constructed despite the lack of doubling condition [4]. Such construction has a continuous counterpart in a class of noncompact Lie groups of exponential growth equipped with flow measures. This is a joint work with De Mari, Levi and Monti [2].

- [1] Christ M., A $T(b)$ theorem with remarks on analytic capacity and the Cauchy integral, Colloq. Math., 1990
- [2] De Mari F., Levi M., Monti M., V. M., Calderón-Zygmund theory on some Lie groups of exponential growth, arXiv:2311.01824
- [3] Grafakos L., Classical Fourier Analysis, 2016
- [4] Levi M., Santagati F., Tabacco A., V.M., Analysis on trees with nondoubling flow measures, Potential Analysis, 2023

Emilio Musso (Politecnico di Torino)

Integrable flows on Legendrian Curves in the 3-dimensional anti-de-Sitter space

We briefly describe the contact structure of type $(1,1)$ on the 3-dimensional anti-de-Sitter space $AdS^{1,2}$. We consider Legendrian curves in $AdS^{1,2}$ satisfying a transversality condition. We show the existence of a family of geometric flows on Legendrian curves related to the KdV hierarchy. At the end, we analyze the evolutions of Legendrian curves originated by the soliton solutions of the KdV equation.

References:

- A. Calini, T. Ivey, E. Musso, mKdV-related flows for Legendrian curves in the pseudo-hermitian 3-sphere, SIGMA, 2024.
- E. Musso, A. Pàmpano, Integrable flows on null curves in the anti-de-Sitter 3-space, arXiv:2311.11137, 2023
- E. Musso, A. Pàmpano, Geometric transformations on null curves in the anti-de-Sitter 3-space, arXiv:2312.10765, 2023.

Claudio Bartocci (Università di Genova)

Moduli spaces of quiver representations

I shall provide a few examples of moduli spaces of quiver representations and discuss their natural geometric structures. Some open questions will be addressed.

Marco Pedroni (Università di Bergamo)

Poisson quasi-Nijenhuis manifolds and Toda lattices

It is well known that the conserved quantities of the open Toda lattice can be described in the geometric framework of Poisson-Nijenhuis manifolds. We will show that the more general notion of Poisson quasi-Nijenhuis manifold can be used to treat the case of the closed Toda lattice. These results were obtained in collaboration with Gregorio Falqui, Igor Mencattini, and Giovanni Ortenzi.