SHARP L^p BOUNDS FOR THE HARDY–LITTLEWOOD MAXIMAL OPERATOR ON GROMOV HYPERBOLIC SPACES

Let (X, d) be a complete, geodesic, Gromov δ-hyperbolic space, and let μ be a positive locally doubling measure on X. We prove that if $1 < a \leq b < a^2$, and the measure of metric balls of radius $r \geq 1$ satisfies

$$ca^r \leq \mu(B_r(x)) \leq Cb^r \qquad \text{for all } x \in X,$$

then the centred Hardy–Littlewood maximal operator is bounded on $L^p(X, \mu)$ for all $p > \tau$ and is of weak type (τ, τ), where $\tau = \log_a b$. Moreover, the index τ is optimal.

Our results apply to Cartan–Hadamard manifolds of pinched negative curvature, providing new boundedness results in these settings.

Based on joint work with Nikolaos Chalmoukis and Stefano Meda.